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Abstract. The plasma mechanisms underlying a recent proposal for setting an upper limit on the mass of the photon are clarified by the study of a simplified model.

1. Introduction

Two years ago an argument was proposed (Williams and Park 1971, to be referred to as WP) by which a very small upper limit could be set for the photon mass (it corresponds to a reduced Compton wavelength $\mu^{-1} = 6$ light year) by showing that the galactic magnetic field would tend to decay in an unacceptably short time if μ were larger than this value. If the assumed model of the galactic field is correct and if no mechanism can be found by which the field could be replenished every 10^6 years (an arbitrary but conservative figure) it follows that an upper limit has been placed on μ. In a recent paper Byrne and Burman (1972) have questioned the argument and stated that when corrected it leads to a larger (and correspondingly less interesting) upper limit for μ. In our opinion the objections are unfounded, but the fact that they could be raised emphasizes the incompleteness of the condensed mathematical discussion given in WP. The aim of this paper is to clarify the argument of WP by developing a visualizable picture of the physical processes involved.

Briefly, the argument in WP runs as follows. The Proca equation (Goldhaber and Nieto 1971) connecting the massive electromagnetic field, the vector potential $\boldsymbol{A}$, and the current source is

$$\nabla \times \boldsymbol{H} - \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} + \mu^2 \boldsymbol{A} = \frac{4\pi}{c}\boldsymbol{j}. \tag{1}$$

If we consider a region in which $\boldsymbol{E}$ is effectively constant and $\boldsymbol{H}$ varies only over distances much greater than μ^{-1} (eventually, distances on a galactic scale), this equation predicts the existence of a current

$$\boldsymbol{j} = \frac{\mu^2 c}{4\pi}\boldsymbol{A} \tag{2}$$

having no counterpart in maxwellian theory. It is the dissipation of energy by this current that ultimately brings down the field.

WP then examined the dissipation of energy in a typical 'cool' galactic cloud, in which only about one hydrogen atom out of 10^3 is ionized, and the presence of the galactic field leads to a spiralling motion of the charged particles present. To set up the equations of motion for the plasma, one assumes that the neutral gas is at rest and that the electronic and ionic currents moving through it undergo friction with the gas with each other characterized by the relaxation times τ_{ea}, τ_{ia}, and τ_{ei} respectively. (In a real cloud the magnetic field causes the charged particles to gyrate through many complete revolutions between collisions with ions or atoms; a justification for representing this motion by a relaxation-time approximation in which they do not gyrate freely at all is given by Chandrasekhar (1960).) If we ignore the forces of inertia, the pressure differential, and the influence of electrons on the ionic motion, the equations of motion for the ions and electrons are

$$e\left(\boldsymbol{E}+\frac{1}{c}\boldsymbol{v}_{\mathrm{i}}\times\boldsymbol{H}\right)-\frac{M}{\tau_{\mathrm{ia}}}\boldsymbol{v}_{\mathrm{i}}=0$$

$$-e\left(\boldsymbol{E}+\frac{1}{c}\boldsymbol{v}_{\mathrm{e}}\times\boldsymbol{H}\right)-\frac{m}{\tau_{\mathrm{ea}}}\boldsymbol{v}_{\mathrm{e}}-\frac{m}{\tau_{\mathrm{ie}}}(\boldsymbol{v}_{\mathrm{e}}-\boldsymbol{v}_{\mathrm{i}})=0$$

where m and M are the electronic and hydrogenic masses. We shall write these for brevity as

$$\boldsymbol{E}+\boldsymbol{v}_{\mathrm{i}}\times\boldsymbol{h}-\alpha\boldsymbol{v}_{\mathrm{i}}=0 \tag{3}$$

$$\boldsymbol{E}+\boldsymbol{v}_{\mathrm{e}}\times\boldsymbol{h}+\beta\boldsymbol{v}_{\mathrm{e}}-\gamma\boldsymbol{v}_{\mathrm{i}}=0 \tag{4}$$

with

$$\alpha=\frac{M}{e\tau_{\mathrm{ia}}},\qquad \beta=\frac{m(\tau_{\mathrm{ie}}^{-1}+\tau_{\mathrm{ea}}^{-1})}{e},\qquad \gamma=\frac{m}{e\tau_{\mathrm{ie}}},\qquad h=\frac{H}{c}.$$

Next we define the current density

$$\boldsymbol{j}=ne(\boldsymbol{v}_{\mathrm{i}}-\boldsymbol{v}_{\mathrm{e}})$$

where n is the number of ions and electrons per unit volume. It is then easy (see Cowling 1957) to show that

$$\boldsymbol{E}=\frac{1}{ne(\alpha+\beta-\gamma)}[\alpha\beta\boldsymbol{j}+(\alpha-\beta)\boldsymbol{j}\times\boldsymbol{h}-(\boldsymbol{j}\times\boldsymbol{h})\times\boldsymbol{h}]. \tag{5}$$

The dissipation of energy is given by

$$\boldsymbol{j}\,.\,\boldsymbol{E}=\frac{1}{ne(\alpha+\beta-\gamma)}[\alpha\beta j^2+(\boldsymbol{j}\times\boldsymbol{h})^2] \tag{6}$$

to which the middle term of (5), representing the plasma Hall effect, makes no contribution. The first term of (6) can be translated into the usual expression for conductivity. Concerning the second term in (6), Byrne and Burman remark that it arises from the interaction between the charged components of the plasma and the neutral substrate, and that since in the astrophysical plasmas considered this interaction is not large, the term should be omitted. This conclusion seems plausible at first glance, though a second glance shows that if there is really no such interaction and τ_{ia} is therefore infinite, we have $\alpha=0$ and the first term vanishes altogether. In fact, the second term is far larger than the first; Cowling (1957) estimates a factor of 10^{11}, while WP (1971) adopted 10^{13}. There is something a little paradoxical in this situation, and since the predominance of the second term of (6) is crucial to the argument for a small upper limit to μ, this paper will consider the matter in more detail.

We have to understand why the small interactions with the substrate produce the predominating effect; this is the more striking when it is noted that in (6),

$$\alpha+\beta-\gamma = \frac{M}{e\tau_{\rm ia}}+\frac{m}{e\tau_{\rm ea}} \tag{7}$$

the larger electron–ion interaction having cancelled. We therefore shall study a simple model to which the above equations apply in order to see how the electronic and ionic currents actually flow and how the dissipation takes place.

2. A simple model

In WP, the galactic arm was represented by the simplified model of a linear column of plasma with a magnetic field along its length (Chandrasekhar and Fermi 1953) and currents flowing transversely to it. Here we shall simplify still further, assuming that the column has circular symmetry about the z axis and that it contains a region in which the current is maintained by a circumferential electric field

$$\boldsymbol{E} = \epsilon \boldsymbol{h}\times\boldsymbol{r} \tag{8}$$

where $\boldsymbol{r}$ is directed normally outward from the z axis. Such an electric field would arise from variations in the magnetic field, and we shall see below that for purposes of this argument it may be arbitrarily small. One effect of E is that it changes the magnetic field as if its flux lines were moving radially: if N is the total flux through a circle of radius r about the z axis, then

$$-\frac{1}{c}\frac{\mathrm{d}N}{\mathrm{d}t} = 2\pi rE = 2\pi r^2\epsilon h = \frac{1}{c}2\pi r^2\epsilon H$$

from which it follows that the effective radial velocity of the flux lines is $\epsilon\boldsymbol{r}$. The other effect of E is to produce currents that dissipate their energy in the plasma.

Equations (3) and (4) can be written

$$(\boldsymbol{v}_{\rm i}-\epsilon\boldsymbol{r})\times\boldsymbol{h} = \alpha\boldsymbol{v}_{\rm i} \tag{9}$$

$$(\boldsymbol{v}_{\rm e}-\epsilon\boldsymbol{r})\times\boldsymbol{h} = -\beta\boldsymbol{v}_{\rm e}+\gamma\boldsymbol{v}_{\rm i}. \tag{10}$$

Let us assume that $h \gg \alpha, \beta, \gamma$, so that the charged particles spiral freely between collisions. Then a zeroth approximation to the solution of (9) and (10) representing motion in the x–y plane is

$$\boldsymbol{v}_{\rm i} = \boldsymbol{v}_{\rm e} = \epsilon\boldsymbol{r} \tag{11}$$

so that electrons and ions travel radially at the same rate as the field lines. The next approximation is found by substituting (11) into the small right-hand sides of (9) and (10) and solving the resulting equations:

$$\boldsymbol{v}_{\rm i} = \epsilon\boldsymbol{r}+\frac{\epsilon\alpha}{h^2}\boldsymbol{h}\times\boldsymbol{r} \tag{12}$$

$$\boldsymbol{v}_{\rm e} = \epsilon\boldsymbol{r}-\frac{\epsilon(\beta-\gamma)}{h^2}\boldsymbol{h}\times\boldsymbol{r}. \tag{13}$$

In this approximation the total electric current density is purely azimuthal,

$$\boldsymbol{j} = \frac{ne\epsilon}{h^2}(\alpha+\beta-\gamma)\boldsymbol{h}\times\boldsymbol{r}. \tag{14}$$

Its magnitude is

$$j = \frac{ne\epsilon}{h}(\alpha+\beta-\gamma)r \tag{15}$$

and the rate of energy dissipation per unit volume is

$$\boldsymbol{E}\,.\,\boldsymbol{j} = ne\epsilon^2(\alpha+\beta-\gamma)r^2. \tag{16}$$

The coefficients α and $\beta-\gamma$ arise only from interactions with the neutral substrate; if they are small the ions and electrons move nearly radially with nearly equal speeds, losing the energy of their directed motions at each collision with a neutral atom.

3. Dissipation in the plasma

We must now see how the decay of the plasma field is governed by the quantities we have derived. The energy density of the massive electromagnetic field is

$$\mathscr{E} = \frac{1}{8\pi}[E^2+H^2+\mu^2(A^2+V^2)] \tag{17}$$

where A and V are the vector and scalar potentials (Goldhaber and Nieto 1971). If the fields cover a region whose linear dimension l is such that $\mu l \gg 1$, the A^2 term dominates (17). Further, the rate of energy dissipation is calculated from (2), (15) and (16) as

$$\boldsymbol{E}\,.\,\boldsymbol{j} = \frac{h^2j^2}{ne(\alpha+\beta-\gamma)} = \left(\frac{\mu^2H}{4\pi}\right)^2\frac{A^2}{ne(\alpha+\beta-\gamma)}$$

where we have reinstated $H = hc$ and note that this relation is independent of the size of the inducing field E. (Thus we may make ϵ very small and our simplified model need not concern itself with the gradual change in density at the centre of the plasma that results from its radial motion, or with the motion of the neutral gas that will occur as a result of its interaction with the moving ions.)

The quantity A^2 decreases by dissipation according to

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\mu^2A^2}{8\pi} = -\left(\frac{\mu^2H}{4\pi}\right)^2\frac{A^2}{ne(\alpha+\beta-\gamma)}$$

and we note that the less the dissipation (the smaller $\alpha+\beta-\gamma$), the more quickly A^2 decreases. The relaxation time τ_A for the decrease of A is estimated by treating H in this equation as a constant, so that

$$\tau_A \simeq \frac{4\pi ne(\alpha+\beta-\gamma)}{\mu^2H^2} = \frac{4\pi n}{\mu^2H^2}\left(\frac{M}{\tau_{\mathrm{ia}}}+\frac{m}{\tau_{\mathrm{ea}}}\right) \tag{18}$$

and since potentials and fields decrease at the same rate, this is also the decay time of the field. This is the result given in WP except that there we omitted the term m/τ_{ea} in (18) (having neglected m in comparison with M throughout) and here we have assumed that $\mu l \gg 1$. If this is not so, μ^2 in (18) becomes μ^2+l^{-2}.

4. Discussion

In order to understand how damping affects the motions of the particles we must first understand what the motions would be in the limit of very small damping. There are two kinds of motion: the first is the usual spiralling of a charged particle in a magnetic field, slightly influenced here by $\boldsymbol{E}$ and the damping.

The second kind of motion is rectilinear at a rate cE/H in crossed fields. This motion is driven by the applied electric field and perists as long as it is applied. It is seen in the limit of small damping in (17) and (18) as an outward motion (if $\epsilon > 0$) at a rate $\epsilon r = cE/H$, which will ultimately be opposed by gas pressure and a radial electric field. The radial drift is the key to the problem. In the absence of friction, the current is entirely radial and gives no contribution to $\boldsymbol{j} . \boldsymbol{E}$. The transverse deflections of the ionic and electronic components are caused by friction, and are responsible for the dissipation of energy. However, the two components move outward with nearly equal speeds, so that the ordinarily large friction between them is here absent. Thus the interactions with the neutral substrate, slight though they may be, are amost entirely responsible for the dissipation of energy.

The model on which we have based our conclusions is not supposed to represent the situation in a real galactic arm (though by specifying the radial dependence of ϵ and h it may be adaptable to such calculations), but physical processes occurring there have their counterparts in the model and are qualitatively explained by the foregoing calculations.

Our conclusions are latent in some of the earlier literature on the subject, especially Cowling (1956) and Mestel and Spitzer (1956), but perhaps our model will clarify the physical mechanisms.

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